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DOUBLE STRONGLY $(r, s)(u, v)$-SEMIOPEN SETS

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Abstract. We introduce the concepts of double strongly $(r, s)(u, v)$-semiopen sets, double strongly $(r, s)(u, v)$-semiclosed sets and double pairwise strongly $(r, s)(u, v)$-semicontinuous mappings in double bitopological spaces and investigate some of their characteristic properties.

1. Introduction

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Çoker and his colleagues [4, 6, 7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [5] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces.

Kandil [8] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces.

In this paper, we introduce the concepts of double strongly $(r, s)(u, v)$-semiopen sets, double strongly $(r, s)(u, v)$-semiclosed sets and double pairwise strongly $(r, s)(u, v)$-semicontinuous mappings in double bitopological spaces and investigate some of their characteristic properties.

2. Preliminaries

Let I be the unit interval $[0, 1]$ of the real line. A member μ of I^X is called a fuzzy set of X. For any $\mu \in I^X$, μ^c denotes the complement $1 - \mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and

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1, respectively. All other notations are standard notations of fuzzy set theory. Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A + \gamma_A \leq \tilde{1}$.

Obviously every fuzzy set μ on X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1} - \mu)$.

An *intuitionistic fuzzy topology* on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties;

(1) $0_\sim, 1_\sim \in T$.
(2) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.
(3) If $A_i \in T$ for all i, then $\bigcup A_i \in T$.

The pair (X, T) is called an *intuitionistic fuzzy topological space.*

Let $I(X)$ be a family of all intuitionistic fuzzy sets of X and let $I \otimes I$ be the set of the pair (r, s) such that $r, s \in I$ and $r + s \leq 1$.

Definition 2.1. [13] Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense* $\mathcal{T}^{\mu\gamma} = (\mathcal{T}^\mu, \mathcal{T}^\gamma)$ on X is a mapping $\mathcal{T}^{\mu\gamma} : I(X) \to I \otimes I(\mathcal{T}^\mu, \mathcal{T}^\gamma : I(X) \to I)$ which satisfies the following properties;

(1) $\mathcal{T}^\mu(0_\sim) = \mathcal{T}^\mu(1_\sim) = 1$ and $\mathcal{T}^\gamma(0_\sim) = \mathcal{T}^\gamma(1_\sim) = 0$.
(2) $\mathcal{T}^\mu(A \cap B) \geq \mathcal{T}^\mu(A) \wedge \mathcal{T}^\mu(B)$ and $\mathcal{T}^\gamma(A \cap B) \leq \mathcal{T}^\gamma(A) \vee \mathcal{T}^\gamma(B)$.
(3) $\mathcal{T}^\mu(\bigcup A_i) \geq \bigwedge \mathcal{T}^\mu(A_i)$ and $\mathcal{T}^\gamma(\bigcup A_i) \leq \bigvee \mathcal{T}^\gamma(A_i)$.

The $(X, \mathcal{T}^{\mu\gamma})$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense.* Also, we call $\mathcal{T}^\mu(A)$ a *gradation of openness* of A and $\mathcal{T}^\gamma(A)$ a *gradation of nonopenness* of A.

Let A be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space in Šostak's sense $(X, \mathcal{T}^{\mu\gamma})$ and $(r, s) \in I \otimes I$. Then A is said to be

(1) a $\mathcal{T}^{\mu\gamma}$*-fuzzy* (r, s)*-open* set if $\mathcal{T}^\mu(A) \geq r$ and $\mathcal{T}^\gamma(A) \leq s$,
(2) a $\mathcal{T}^{\mu\gamma}$*-fuzzy* (r, s)*-closed* set if $\mathcal{T}^\mu(A^C) \geq r$ and $\mathcal{T}^\gamma(A^C) \leq s$.

Let $(X, \mathcal{T}^{\mu\gamma})$ be an intuitionistic fuzzy topological space in Šostak's sense. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the $\mathcal{T}^{\mu\gamma}$*-fuzzy* (r, s)*-closure* is defined by

$$\mathcal{T}^{\mu\gamma}\text{-cl}(A, (r, s)) = \bigcap \{B \in I(X) \mid A \subseteq B, B \text{ is } \mathcal{T}^{\mu\gamma}\text{-fuzzy } (r, s)\text{-closed}\}$$

and the $\mathcal{T}^{\mu\gamma}$-*fuzzy* (r,s)-*interior* is defined by

$$\mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s)) = \bigcup\{B \in I(X) \mid A \supseteq B, B \text{ is } \mathcal{T}^{\mu\gamma}\text{-fuzzy } (r,s)\text{-open}\}.$$

LEMMA 2.2. [10] For an intuitionistic fuzzy set A in an intuitionistic fuzzy topological space in Šostak's sense $(X,\mathcal{T}^{\mu\gamma})$ and $(r,s) \in I \otimes I$, we have

(1) $\mathcal{T}^{\mu\gamma}\text{-cl}(A,(r,s))^C = \mathcal{T}^{\mu\gamma}\text{-int}(A^C,(r,s))$, and
(2) $\mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s))^C = \mathcal{T}^{\mu\gamma}\text{-cl}(A^C,(r,s))$.

A system $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ consisting of a set X with two intuitionistic fuzzy topologies in Šostak's sense $\mathcal{T}^{\mu\gamma}$ and $\mathcal{U}^{\mu\gamma}$ on X is called a *double bitopological space.*

DEFINITION 2.3. [10, 11] Let A be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v) \in I \otimes I$. Then A is said to be

(1) a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-*double* $(r,s)(u,v)$-*semiopen* set if there is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open set B in X such that $B \subseteq A \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(B,(u,v))$,
(2) a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-*double* $(r,s)(u,v)$-*semiclosed* set if there is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-closed set B in X such that $\mathcal{U}^{\mu\gamma}\text{-int}(B,(u,v)) \subseteq A \subseteq B$,
(3) a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-*double* $(r,s)(u,v)$-*preopen* set if

$$A \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(A,(u,v)),(r,s)),$$

(4) a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-*double* $(r,s)(u,v)$-*preclosed* set if

$$A \supseteq \mathcal{T}^{\mu\gamma}\text{-cl}((\mathcal{U}^{\mu\gamma}\text{-int}(A,(u,v)),(r,s)).$$

3. Double strongly $(r,s)(u,v)$-semiopen sets

DEFINITION 3.1. Let A be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v) \in I \otimes I$. Then A is said to be

(1) a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-*double strongly* $(r,s)(u,v)$-*semiopen* set if there is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open set B in X such that

$$B \subseteq A \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(B,(u,v)),(r,s)),$$

(2) a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-*double strongly* $(u,v)(r,s)$-*semiopen* set if there is a $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-open set B in X such that

$$B \subseteq A \subseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(B,(r,s)),(u,v)),$$

(3) a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-*double strongly* $(r,s)(u,v)$-*semiclosed* set if there is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-closed set B in X such that

$$\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(B,(u,v)),(r,s)) \subseteq A \subseteq B,$$

(4) a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-*double strongly* $(u,v)(r,s)$-*semiclosed* set if there is a $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-closed set B in X such that

$$\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(B,(r,s)),(u,v)) \subseteq A \subseteq B.$$

THEOREM 3.2. Let A be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v) \in I \otimes I$. Then the following statements are equivalent;

(1) A is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiopen set.
(2) A^C is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiclosed set.
(3) $A \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s)),(u,v)),(r,s))$.
(4) $A^C \supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(A^C,(r,s)),(u,v)),(r,s))$.
(5) A is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen and $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen set.
(6) A^C is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiclosed and $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set.

Proof. (1) $\Leftrightarrow$ (2), (3) $\Leftrightarrow$ (4) and (5) $\Leftrightarrow$ (6) follow from Lemma 2.2.

(1) $\Rightarrow$ (3) Let A be a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiopen set. Then there is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open set B in X such that $B \subseteq A \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(B,(u,v)),(r,s))$. Since $B \subseteq A$, we have

$$B = \mathcal{T}^{\mu\gamma}\text{-int}(B,(r,s)) \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s)).$$

Hence

$$\begin{aligned} A &\subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(B,(u,v)),(r,s)) \\ &\subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s)),(u,v)),(r,s)). \end{aligned}$$

(3) $\Rightarrow$ (1) Let $A \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s)),(u,v)),(r,s))$. Suppose that $B = \mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s))$. Then B is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open set and

$$\begin{aligned} B = \mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s)) &\subseteq A \\ &\subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s)),(u,v)),(r,s)) \\ &= \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(B,(u,v)),(r,s)). \end{aligned}$$

Thus A is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiopen set.

(1) $\Rightarrow$ (5) It is obvious.

(5) $\Rightarrow$ (3) Let A be a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen and $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double preopen set of X. Then $A \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s)),(u,v))$ and $A \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(A,(u,v)),(r,s))$. Therefore

$$\begin{aligned} A &\subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(A,(u,v)),(r,s)) \\ &\subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s)),(u,v)),(u,v)),(r,s)) \\ &= \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s)),(u,v)),(r,s)). \end{aligned}$$

This completes the proof. □

Corollary 3.3. Let A be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v) \in I \otimes I$. Then the following statements are equivalent;

(1) A is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiopen set.
(2) A^C is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiclosed set.
(3) $A \subseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(A,(u,v)),(r,s)),(u,v))$.
(4) $A^C \supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(A^C,(u,v)),(r,s)),(u,v))$.
(5) A is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiopen and $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen set.
(6) A^C is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiclosed and $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed set.

It is clear that every $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open ((r,s)-closed) set is $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiopen (strongly $(r,s)(u,v)$-semiclosed) and every $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-open ((u,v)-closed) set is $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiopen (strongly $(u,v)(r,s)$-semiclosed) but the converse need not be true which is shown by the following example.

Example 3.4. Let $X = \{x,y\}$ and let A_1, A_2, A_3 and A_4 be intuitionistic fuzzy sets of X defined as

$$A_1(x) = (0.1, 0.8), \quad A_1(y) = (0.3, 0.2);$$

$$A_2(x) = (0.5, 0.3), \quad A_2(y) = (0.6, 0.1);$$

$$A_3(x) = (0.2, 0.5), \quad A_3(y) = (0.7, 0.1);$$

and

$$A_4(x) = (0.6, 0.2), \quad A_4(y) = (0.8, 0.1).$$

Define $\mathcal{T}^{\mu\gamma} : I(X) \to I \otimes I$ and $\mathcal{U}^{\mu\gamma} : I(X) \to I \otimes I$ by

$$\mathcal{T}^{\mu\gamma}(A) = (\mathcal{T}^{\mu}(A), \mathcal{T}^{\gamma}(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{5}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}^{\mu\gamma}(A) = (\mathcal{U}^{\mu}(A), \mathcal{U}^{\gamma}(A)) = \begin{cases} (1,0) & \text{if } A = 0_{\sim}, 1_{\sim}, \\ (\frac{1}{3}, \frac{1}{4}) & \text{if } A = A_2, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ is a double bitopological space on X. The intuitionistic fuzzy set A_3 is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double strongly $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-semiopen set which is not a $\mathcal{T}^{\mu\gamma}$-fuzzy $(\frac{1}{2}, \frac{1}{5})$-open set and A_3^C is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double strongly $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-semiclosed set which is not a $\mathcal{T}^{\mu\gamma}$-fuzzy $(\frac{1}{2}, \frac{1}{5})$-closed set. Also A_4 is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double strongly $(\frac{1}{3}, \frac{1}{4})(\frac{1}{2}, \frac{1}{5})$-semiopen set which is not a $\mathcal{U}^{\mu\gamma}$-fuzzy $(\frac{1}{3}, \frac{1}{4})$-open set and A_4^C is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double strongly $(\frac{1}{3}, \frac{1}{4})(\frac{1}{2}, \frac{1}{5})$-semiclosed set which is not a $\mathcal{U}^{\mu\gamma}$-fuzzy $(\frac{1}{3}, \frac{1}{4})$-closed set.

THEOREM 3.5. Let $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ be a double bitopological space and $(r,s), (u,v) \in I \otimes I$.

(1) If $\{A_k\}$ is a family of $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiopen sets of X, then $\cup A_k$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiopen set.

(2) If $\{A_k\}$ is a family of $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiopen sets of X, then $\cup A_k$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiopen set.

(3) If $\{A_k\}$ is a family of $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiclosed sets of X, then $\cap A_k$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiclosed set.

(4) If $\{A_k\}$ is a family of $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiclosed sets of X, then $\cap A_k$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiclosed set.

Proof. (1) Let $\{A_k\}$ be a collection of $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiopen sets. Then for each k, there is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open set B_k such that $B_k \subseteq A_k \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(B_k, (u,v)), (r,s))$. Since B_k is $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open, $\mathcal{T}^{\mu}(B_k) \geq r$ and $\mathcal{T}^{\gamma}(B_k) \leq s$ for each k. So $\mathcal{T}^{\mu}(\cup B_k) \geq \wedge \mathcal{T}^{\mu}(B_k) \geq r$ and $\mathcal{T}^{\gamma}(\cup B_k) \leq \vee \mathcal{T}^{\gamma}(B_k) \leq s$. Thus $\cup B_k$ is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open set. Also, we have

$$\begin{aligned} \cup B_k \subseteq \cup A_k &\subseteq \cup \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(B_k, (u,v)), (r,s)) \\ &\subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(\cup B_k, (u,v)), (r,s)). \end{aligned}$$

Hence $\cup A_k$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiopen set.

(2) Let $\{A_k\}$ be a collection of $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiopen sets. Then for each k, there is a $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-open set B_k

such that $B_k \subseteq A_k \subseteq \mathcal{U}^{\mu\gamma}$-int$(\mathcal{T}^{\mu\gamma}$-cl$(B_k,(r,s)),(u,v))$. Since B_k is $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-open, $\mathcal{U}^{\mu}(B_k) \geq u$ and $\mathcal{U}^{\gamma}(B_k) \leq v$ for each k. So $\mathcal{U}^{\mu}(\cup B_k) \geq \wedge\mathcal{U}^{\mu}(B_k) \geq u$ and $\mathcal{U}^{\gamma}(\cup B_k) \leq \vee\mathcal{U}^{\gamma}(B_k) \leq v$. Thus $\cup B_k$ is a $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-open set. Also, we have

$$\begin{aligned}\cup B_k \subseteq \cup A_k &\subseteq \cup\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(B_k,(r,s)),(u,v))\\ &\subseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(\cup B_k,(r,s)),(u,v)).\end{aligned}$$

Hence $\cup A_k$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiopen set.

(3) It follows from (1) and Theorem 3.2.

(4) It follows from (2) and Corollary 3.3. □

It is obvious that every $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiopen $((\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiopen) set is not only a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen $((\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiopen) set but also a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen $((\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen) set. However, the following example show that all of the converses need not be true.

Example 3.6. Let $X=\{x,y\}$ and let A_1,A_2,A_3,A_4,A_5 and A_6 be intuitionistic fuzzy sets of X defined as

$$\begin{aligned}A_1(x)&=(0.1,0.6), & A_1(y)&=(0.5,0.4);\\ A_2(x)&=(0.5,0.2), & A_2(y)&=(0.3,0.6);\\ A_3(x)&=(0.2,0.6), & A_3(y)&=(0.5,0.3);\\ A_4(x)&=(0.1,0.6), & A_4(y)&=(0.4,0.5);\\ A_5(x)&=(0.5,0.1), & A_5(y)&=(0.4,0.5);\end{aligned}$$

and

$$A_6(x)=(0.4,0.5),\quad A_6(y)=(0.2,0.7).$$

Define $\mathcal{T}^{\mu\gamma}: I(X)\to I\otimes I$ and $\mathcal{U}^{\mu\gamma}: I(X)\to I\otimes I$ by

$$\mathcal{T}^{\mu\gamma}(A)=(\mathcal{T}^{\mu}(A),\mathcal{T}^{\gamma}(A))=\begin{cases}(1,0) & \text{if } A=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{5}) & \text{if } A=A_1,\\ (0,1) & \text{otherwise;}\end{cases}$$

and

$$\mathcal{U}^{\mu\gamma}(A)=(\mathcal{U}^{\mu}(A),\mathcal{U}^{\gamma}(A))=\begin{cases}(1,0) & \text{if } A=0_\sim,1_\sim,\\ (\frac{1}{3},\frac{1}{4}) & \text{if } A=A_2,\\ (0,1) & \text{otherwise.}\end{cases}$$

Then clearly $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ is a double bitopological space on X. The intuitionistic fuzzy set A_3 is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(\frac{1}{2},\frac{1}{5})(\frac{1}{3},\frac{1}{4})$-semiopen set which is not a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double strongly $(\frac{1}{2},\frac{1}{5})(\frac{1}{3},\frac{1}{4})$-semiopen set and A_4 is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(\frac{1}{2},\frac{1}{5})(\frac{1}{3},\frac{1}{4})$-preopen set which is not a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double strongly $(\frac{1}{2},\frac{1}{5})(\frac{1}{3},\frac{1}{4})$-semiopen set. Also A_5 is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(\frac{1}{3},\frac{1}{4})(\frac{1}{2},\frac{1}{5})$-semiopen set which is not a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double strongly $(\frac{1}{3},\frac{1}{4})(\frac{1}{2},\frac{1}{5})$-semiopen set and A_6 is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(\frac{1}{3},\frac{1}{4})(\frac{1}{2},\frac{1}{5})$-preopen set which is not a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double strongly $(\frac{1}{3},\frac{1}{4})$ $(\frac{1}{2},\frac{1}{5})$-semiopen set.

Definition 3.7. Let $f:(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\to(Y,\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})$ be a mapping from a double bitopological space X to a double bitopological space Y and $(r,s),(u,v)\in I\otimes I$. Then f is called a *double pairwise strongly* $(r,s)(u,v)$*-semicontinuous* mapping if $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiopen set of X for each $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-open set A of Y and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiopen set of X for each $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-open set B of Y.

Theorem 3.8. Let $f:(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\to(Y,\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})$ be a mapping and $(r,s),(u,v)\in I\otimes I$. Then the following statements are equivalent;

(1) f is a double pairwise strongly $(r,s)(u,v)$-semicontinuous mapping.

(2) $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiclosed set of X for each $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-closed set A of Y and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiclosed set of X for each $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-closed set B of Y.

(3) For each intuitionistic fuzzy set A of Y,
$$\begin{aligned}&\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(A),(r,s)),(u,v)),(r,s))\\&\subseteq f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A,(r,s)))\end{aligned}$$
and
$$\begin{aligned}&\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(A),(u,v)),(r,s)),(u,v))\\&\subseteq f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A,(u,v))).\end{aligned}$$

(4) For each intuitionistic fuzzy set C of X,
$$\begin{aligned}&f(\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(C,(r,s)),(u,v)),(r,s)))\\&\subseteq \mathcal{V}^{\mu\gamma}\text{-cl}(f(C),(r,s))\end{aligned}$$
and
$$\begin{aligned}&f(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(C,(u,v)),(r,s)),(u,v)))\\&\subseteq \mathcal{W}^{\mu\gamma}\text{-cl}(f(C),(u,v)).\end{aligned}$$

Proof. (1) $\Rightarrow$ (2) Let A be any $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-closed set and B any $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-closed set of Y. Then A^C is a $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-open set and B^C is a $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-open set of Y. Since f is a double pairwise strongly $(r,s)(u,v)$-semicontinuous mapping, $f^{-1}(A^C)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiopen set and $f^{-1}(B^C)$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiopen set of X. By Theorem 3.2 and Corollary 3.3, $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiclosed set and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiclosed set of X.

(2) $\Rightarrow$ (1) Let A be any $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-open set and B any $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-open set of Y. Then A^C is a $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-closed set and B^C is a $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-closed set of Y. By (2), $f^{-1}(A^C)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiclosed set and $f^{-1}(B^C)$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiclosed set of X. By Theorem 3.2 and Corollary 3.3, $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiopen set and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiopen set of X. Thus f is a double pairwise strongly $(r,s)(u,v)$-semicontinuous mapping.

(2) $\Rightarrow$ (3) Let A be any intuitionistic fuzzy set of Y. Then $\mathcal{V}^{\mu\gamma}$-cl$(A,(r,s))$ is a $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-closed set and $\mathcal{W}^{\mu\gamma}$-cl$(A,(u,v))$ is a $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-closed set of Y. By (2), $f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A,(r,s)))$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiclosed set and $f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A,(u,v)))$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiclosed set of X. Thus

$$\begin{aligned}
&f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A,(r,s)))\\
&\supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A,(r,s))),(r,s)),(u,v)),(r,s))\\
&\supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(A),(r,s)),(u,v)),(r,s))
\end{aligned}$$

and

$$\begin{aligned}
&f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A,(u,v)))\\
&\supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A,(u,v))),(u,v)),(r,s)),(u,v))\\
&\supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(A),(u,v)),(r,s)),(u,v)).
\end{aligned}$$

(3) $\Rightarrow$ (4) Let C be any intuitionistic fuzzy set of X. Then $f(C)$ is an intuitionistic fuzzy set of Y. By (3)

$$\begin{aligned}
&f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(f(C),(r,s)))\\
&\supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}f(C),(r,s)),(u,v)),(r,s))\\
&\supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(C,(r,s)),(u,v)),(r,s))
\end{aligned}$$

and

$$\begin{aligned}&f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(f(C),(u,v)))\\&\supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}f(C),(u,v)),(r,s)),(u,v))\\&\supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(C,(u,v)),(r,s)),(u,v)).\end{aligned}$$

Hence

$$\begin{aligned}\mathcal{V}^{\mu\gamma}\text{-cl}(f(C),(r,s)) &\supseteq ff^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(f(C),(r,s)))\\&\supseteq f(\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(C,(r,s)),(u,v)),(r,s)))\end{aligned}$$

and

$$\begin{aligned}\mathcal{W}^{\mu\gamma}\text{-cl}(f(C),(u,v)) &\supseteq ff^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(f(C),(u,v)))\\&\supseteq f(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(C,(u,v)),(r,s)),(u,v))).\end{aligned}$$

(4) $\Rightarrow$ (2) Let A be any $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-closed set and B any $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-closed set of Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are intuitionistic fuzzy sets of X. By (4),

$$\begin{aligned}&f(\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(A),(r,s)),(u,v)),(r,s)))\\&\subseteq \mathcal{V}^{\mu\gamma}\text{-cl}(ff^{-1}(A),(r,s))\\&\subseteq \mathcal{V}^{\mu\gamma}\text{-cl}(A,(r,s))=A\end{aligned}$$

and

$$\begin{aligned}&f(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(B),(u,v)),(r,s)),(u,v)))\\&\subseteq \mathcal{W}^{\mu\gamma}\text{-cl}(ff^{-1}(B),(u,v))\\&\subseteq \mathcal{W}^{\mu\gamma}\text{-cl}(B,(u,v))=B.\end{aligned}$$

So we have

$$\begin{aligned}&\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(A),(r,s)),(u,v)),(r,s))\\&\subseteq f^{-1}f(\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(A),(r,s)),(u,v)),(r,s)))\\&\subseteq f^{-1}(A)\end{aligned}$$

and

$$\begin{aligned}&\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(B),(u,v)),(r,s)),(u,v))\\&\subseteq f^{-1}f(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(B),(u,v)),(r,s)),(u,v)))\\&\subseteq f^{-1}(B).\end{aligned}$$

Thus $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiclosed set and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiclosed set of X. $\square$

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